ERROR ESTIMATION FOR THE FEM SOLUTION WITH A FEW BAD ELEMENTS

KENTA KOBAYASHI

Abstract. Let Ω be a convex polygonal domain. For $f \in L^2(\Omega)$, we consider P^1 finite element solution of the following Poisson equation:

$$\begin{cases} -\Delta u = f & \text{in} \quad \Omega \\ \quad u = 0 & \text{on} \quad \partial\Omega \end{cases}.$$

In conventional error analysis for the finite element method, typical error estimation for the finite element solution u_h is obtained by the following form [1]:

$$|u_h - u|_{H^1(\Omega)} \leq \max_{\tau_j} C(\tau_j)|u|_{H^2(\Omega)}$$

where H^1 semi-norm and H^2 semi-norm are defined as follows

$$|v|^2_{H^1(\Omega)} = |v_x|^2_{L^2(\Omega)} + |v_y|^2_{L^2(\Omega)}, \qquad |v|^2_{H^2(\Omega)} = |v_{xx}|^2_{L^2(\Omega)} + 2|v_{xy}|^2_{L^2(\Omega)} + |v_{yy}|^2_{L^2(\Omega)},$$

τ_j are triangular elements which composing a mesh division, and $C(T)$ is a constant which only depends on triangle T.

Concerning this error estimation, even one bad element results in poor error estimation because we take maximum of $C(\tau_j)$. However, numerical results suggest that a few bad elements do not lead to an increase in error. We have provided theoretical proof for this fact with the concrete error estimation. In other words, under certain conditions, we proved that the presence of a few bad elements does not worsen the error of the finite element method.

Keywords: finite element method, error estimation, triangular elements

Mathematics Subject Classifications (2010): 65N30

References

[1] S. C. Brenner , L. R. Scott. *The Mathematical Theory of Finite Element Methods*, Springer, New York, 2008.

Hitotsubashi University, Japan
Email address: `kenta.k@r.hit-u.ac.jp`