POLYNOMIAL–DEGREE–ROBUST A POSTERIORI ESTIMATES FOR THE VIRTUAL ELEMENT METHOD

THÉOPHILE CHAUMONT-FRELET, JOSCHA GEDICKE, AND LORENZO MASCOTTO

ABSTRACT. The virtual element method (VEM) is becoming increasingly popular to discretize PDEs due to its ability to operate on general polytopal meshes and its structure-preserving properties [1]. To reliably assess the error associated with the VEM approximation and efficiently adapt the mesh, *a posteriori* error estimators have been proposed in the literature; see, e.g., [2, 3, 4]. The estimators developed therein are residual-based and have two shortcomings: the constants in the inequalities linking the estimator to the error depend on (i) the polynomial degree of the discretization and (ii) the choice of stabilization. In contrast, for standard finite element (FEM) and discontinuous Galerkin discretizations, equilibrated estimators have been proposed to alleviate these limitations [7]. Although the framework proposed in [7] is fairly general, it cannot be immediately applied to the VEM, due to the nature of the stabilization and the fact that the shape functions are virtual [6]. In this talk, I will present *a posteriori* error estimates that are robust with respect to the polynomial degree and the choice of stabilization [5]. Beyond its intrinsic interest, the development of the estimator introduces new tools that may be helpful for other aspects of the numerical analysis of the VEM.

Keywords: a posteriori error estimates; flux-equilibration; high-order methods, virtual element methods.

Mathematics Subject Classifications (2010): 65L60, 65N15.

REFERENCES

[1] L. Beirão da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L.D. Marini, and A. Russo. Basic principles of virtual element methods. *Mathematical Models and Methods in Applied Sciences*, 23(1):199-214, 2013.

[2] L. Beirão da Veiga and G. Manzini. Residual a posteriori error estimation for the virtual element method for elliptic problems. ESAIM *Mathematical Modelling and Numerical Analysis*, 49(2):577-599, 2015.

[3] S. Berrone and A. Borio. A residual a posteriori error estimate for the virtual element method. *Mathematical Models and Methods in Applied Sciences*, 27(8), 1423-1458, 2007.

[4] A. Cangiani, E.H. Georgoulis, T. Pryer, and O.J. Sutton. A posteriori error estimates for the virtual element method. *Numerische Mathematik*, 137(4):857-893, 2017.

[5] T. Chaumont-Frelet, J. Gedicke and L. Mascotto. Generalized gradients for virtual elements and applications to a posteriori error analysis. *In preparation*, 2023.

[6] F. Dassi, J. Gedicke and L. Mascotto. Adaptive virtual element methods with equilibrated fluxes. *Applied Numerical Mathematics*, 173:249-278, 2022.

[7] A. Ern and M. Vohralík. Polynomial-degree-robust a posteriori estimates in a unified setting for conforming, nonconforming, discontinuous Galerkin, and mixed discretizations. SIAM *Journal on Numerical Analysis*, 53(2):1058-1081, 2015.

INRIA, UNIV. LILLE, CNRS, UMR 8524 – LABORATOIRE PAUL PAINLEVÉ
Email address: theophile.chaumont@inria.fr

INSTITUT FÜR NUMERISCHE SIMULATION, UNIVERSITÄT BONN
Email address: gedicke@ins.uni-bonn.de

DIPARTIMENTO DI MATEMATICA E APPLICAZIONI, UNIVERSITÀ DI MILANO BICOCCA
Email address: lorenzo.mascotto@unimib.it