NUMERICAL SIMULATION OF PHASE CHANGE PROBLEMS BY VARIATIONAL MULTISCALE TECHNIQUES

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Abstract. Phase change problems are processes that consists in the change of state of the matter from liquid to solid as a result of decreasing temperature. Its understanding is a critical step in applied engineering design in several industry areas such as the freezing of foods to preserve their quality and the solidification of pure metals and alloys to obtaing high quality products. The mathematical model for this kind of problems consist in a coupled non-linear system of partial differential equations including continuity, linear momentum and energy. More specifically, if $t_f > 0$ is a time horizon and $\Omega \subset \mathbb{R}^d, d = 2, 3$ is a bounded domain with boundary $\partial\Omega$, we want to find the velocity $\boldsymbol{u}$, pressure p, temperature T and the phase change function $f_{pc}(T)$ such that:

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho\boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nabla\cdot(2\mu\nabla^s\boldsymbol{u}) + \mathcal{K}(f_{pc}(T))\boldsymbol{u} + \nabla p = \boldsymbol{f}(\rho, T), \quad \text{in } (0, t_f)\times\Omega, \tag{1}$$

$$\nabla\cdot\boldsymbol{u} = 0, \quad \text{in } (0, t_f)\times\Omega, \tag{2}$$

$$\left[\rho C_p + L\frac{\partial f_{pc}(T)}{\partial T}\right]\left[\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T\right] = \nabla\cdot(\kappa\nabla T), \quad \text{in } (0, t_f)\times\Omega. \tag{3}$$

This system is complemented with suitable initial and boundary conditions. Additionally, $\nabla^s\boldsymbol{u} := \frac{1}{2}(\nabla\boldsymbol{u} + (\nabla\boldsymbol{u})^T)$ is the symmetric gradient of the velocity, $\boldsymbol{f}(\rho, T)$ is the body force vector, μ is the viscosity, κ is the thermal conductivity, ρ is the density, C_p is the apparent specific heat, L is the latent heat, $\mathcal{K}(f_{pc}(T))$ is a function used to reduce the velocity in the solidifying zone [ref].

In this talk, we present a numerical algorithm to solve system (**??**)-(**??**) based on a variational multiscale finite element method introduced by Hughes [**?**]. The method extend the ideas presented by Castillo and Codina in [**?**] performing a stabilization term-by-term with dynamic subescales for the velocity, pressure and temperature. We perform numerical simulations of some problems involving water freezing [**?**] and alloy solidification [**?**], to show the robustness and the efficiency of the proposed numerical scheme.

Keywords: Phase change, Finite element method, Variational multiscale framework.

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