# A FINITE ELEMENT METHOD FOR THE NAVIER-STOKES EQUATION WITH FREE SURFACE

GABRIEL R. BARRENECHEA

Abstract. in this talk I will present a new finite element method for the incompressible Navier-Stokes equations with free surface. The evolution of the free surface is driven by the kinematic boundary condition, and an Arbitrary Lagrangian Eulerian (ALE) approach is used to derive a (formal) weak formulation which involves three fields, namely, velocity, pressure, and the function describing the free surface. This formulation is discretised using finite elements in space and a time-advancing explicit finite difference scheme in time. In fact, the domain tracking algorithm is explicit: first, we solve the equation for the free surface, then move the mesh according to the sigma transform, and finally we compute the velocity and pressure in the updated domain. This explicit strategy is built in such a way that global conservation can be proven, which plays a pivotal role in the proof of stability of the discrete problem. In addition, in the first and third steps of this algorithms appropriate stabilisation terms are added in order to prove stability of the scheme. The well-posedness and stability results are independent of the viscosity of the fluid, but while the proof of stability for the velocity is valid for all time steps, and all geometries, the stability for the free surface requires a CFL condition. The performance of the current approach is presented via numerical results and comparisons with the characteristics finite element method.

The work presented in this talk has been done in collaboration with Emmanuel Audusse (Université Sorbonne Paris Nord), Astrid Decoene (Université de Bordeaux), and Pierrick Quemar (Université Sorbonne Paris Nord).

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University of Strathclyde
*Email address*: gabriel.barrenechea@strath.ac.uk