EFFICIENT SOLVERS IN NONLINEAR POROELASTICITY

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ABSTRACT. Poroelasticity addresses the deformation of a porous medium, understood as a continuum with a complex interconnected structure through which fluid flows. This field has become attractive from a modeling perspective in biomedicine because soft tissue is naturally porous, as it contains both the circulatory and lymphatic systems. Its downside is that the equations describing such phenomena are much more involved than their elastic conterparts, which makes them more difficult to handle. A very successful strategy for solving the equations of poroelasticity, in the linear setting, has been that of alternating minimization methods, which applied to the original problem gives rise to the *undrained* scheme, whereas the dual formulation gives rise to the *fixed-stress* scheme. The connection of these methods to alternate minimization was possible after finding the mathematical structure of poroelasticity: Generalized Gradient Flows [1].

In this talk, we will study the mathematical structure of nonlinear poroelasticity, which has the main difficulty of presenting pressure as a function of porosity and not as a variable. This work extends previous works on linear and partially nonlinear poroelasticity [2, 3]. Its main objective of this study is that of developing robust iterative methods for poroelasticity, where an interesting feature of the model is that a primal formulation is *never* consistent, as it relies on second order derivatives that are not well-defined in the standard functional setting. We refer to this problem as *primal inconsistency*. Our formulation will allow for an iterative procedure that leverages the theory of alternating minimizations, and at the same time is robust against the primal inconsistency phenomenon. The resulting solvers are robust and optimal, as we will show through numerical tests for up to several millions of degrees of freedom. This guarantees that our methodology is transparently transferable to supercomputing infrastructures.

Keywords: Porous media, nonlinear poroelasticity, scalable solvers, alternate minimization

Mathematics Subject Classifications (2010): 65F10, 74S05, 76S05

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