TENSOR NETWORKS SPACE-TIME SPECTRAL ELEMENTS METHOD FOR SOLVING HEAT EQUATION

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Abstract. In this work, we have coupled Spectral Methods with tensor train format for the efficient low- rank approximation of the multi-dimensional elliptic equation. For the discretization of the model problem, we have employed collocation spectral methods, which are justified for applying the tensor train format. We propose two different discretization for the global space-time formulation, resulting in a large block linear system, encapsulating all time steps, and solve it at once in the TT/QTT-formats. The linear complexity of storage and the solution time is observed in both spatial and time grid sizes. Further, we develop the theory without assuming the separable condition of the force function, which leads to function train decomposition of the force function. In addition, we have studied the cross-tensor train decomposition for the general solution while avoiding the rank-1 assumption of the solution. The method is applied to the Richards equation arising from the unsaturated water flow in the soil.

Keywords: Multi-dimensional PDEs, Tensor-Train Format, Quantized Tensor-Train Format, Richards Equations.

Mathematics Subject Classifications (2010): 35K20, 65F50, 15A69, 65D15, 33F05, 65F10

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