AN IMEX-RK SCHEMES FOR NONLOCAL EQUATIONS WITH A GRADIENT FLOW STRUCTURE IN 1D

RAIMUND BÜRGER, DANIEL INZUNZA, PEP MULET, AND LUIS MIGUEL VILLADA

Abstract. Nonlinear convection-diffusion equations with nonlocal flux and possibly degenerate diffusion arise in various contexts including interacting gases, porous media flows, collective behavior in biology, crowd dynamics and traffic flows. Their numerical solution by an explicit finite difference method is costly due to the necessity of discretizing a local spatial convolution for each evaluation of the convective numerical flux, and due to the disadvantageous Courant-Friedrichs-Lewy (CFL) condition incurred by the diffusion term. Based on explicit schemes for such models devised in [1] a second-order implicit-explicit Runge-Kutta (IMEX-RK) method can be formulated. This method avoids the restrictive time step limitation of explicit schemes since the diffusion term is handled implicitly, but entails the necessity to solve nonlinear algebraic systems in every time step. It is proved that this method is well defined. Numerical experiments illustrate that for fine discretizations it is more efficient in terms of reduction of error versus CPU time than the original explicit method. One of the test cases is given by a strongly degenerate parabolic, nonlocal equation modelling aggregation [2]. This model can be transformed to a local partial differential equation that can be solved numerically easily to generate a reference solution for the IMEX-RK method, but is limited to one space dimension.

References

[1] J.A. Carrillo, A. Chertock, Y. Huang. A finite-volume method for nonlinear nonlocal equations with a gradient flow structure. *Commun. Comput. Phys.*, vol. 17 (2015) pp. 233–258.

[2] F. Betancourt, R. Bürger, K.H. Karlsen. A strongly degenerate parabolic aggregation equation. *Commun. Math. Sci.* vol. 9 (2011) pp. 711–742.

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Chile
E-mail address: rburger@ing-mat.udec.cl

CI^{2}MA and Departamento de Ingeniería Matemática, Universidad de Concepción, Chile
E-mail address: dinzunza@ing-mat.udec.cl

Departament de Matemàtiques, Universitat de València, Spain
E-mail address: mulet@uv.es

CI^{2}MA and Departamento de Matemáticas, Universidad del Bío-Bío, Chile
E-mail address: lvillada@ubb.cl