EQUIVALENCE OF LOCAL- AND GLOBAL-BEST APPROXIMATIONS AND SIMPLE STABLE LOCAL COMMUTING PROJECTORS IN $\boldsymbol{H}(\mathrm{div},\Omega)$

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Abstract. We prove, under minimal regularity, results permitting to localize the global-best approximations by $\boldsymbol{H}(\mathrm{div},\Omega)$-conforming finite element spaces. More precisely, we show that the global best-approximation of a given $\boldsymbol{H}(\mathrm{div},\Omega)$ function in a $\boldsymbol{H}(\mathrm{div},\Omega)$-conforming finite element space imposing the normal trace continuity constraint can be bounded above and below by the sum of the respective local approximations from the elementwise local spaces without any inter-element continuity imposed along the interfaces. Applications of these results leading to optimal a priori error estimates that avoid any notion of an interpolation operator and apply under minimal regularity to mixed finite element and least-squares element methods are presented. Incidentally, we construct a projector from $\boldsymbol{H}(\mathrm{div},\Omega)$ to its conforming finite element subspace that is simultaneously locally defined (over patches of elements), simple as obtained via local-best approximation followed by "flux reconstruction", commuting with the $L^2(\Omega)$-projection, and stable in the $L^2(\Omega)$ norm. These results are described in [1] in extension of our previous contribution [2] and building upon [3].

Keywords: local-best approximation, global-best approximation, equivalence of approximations, $\boldsymbol{H}(\mathrm{div},\Omega)$-conforming elements, mixed methods, least square methods, minimal regularity, optimal estimates

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