A BALANCING DOMAIN DECOMPOSITION METHOD INCORPORATING VIRTUAL ELEMENTS FOR MAGNETOSTATIC PROBLEMS

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Abstract. A Balancing Domain Decomposition (BDD) method incorporating Virtual Elements (VE) is considered as a preconditioner for the iterative procedure in the Hierarchical Domain Decomposition Methods (HDDM) for magnetostatic problems.

HDDM has enabled us to execute large-scale computational models of magnetostatic, eddy current, and high-frequency electromagnetic problems whose numbers of Degrees Of Freedom (DOF) are about 10^7–10^9; see, for example, Kanayama, et al. [2] and Sugimoto, et al. [5]. HDDM does not explicitly construct the resultant linear system derived from a finite element approximation of the interface problem, and replaces the matrix-vector product in the iterative procedure to solve the resultant linear one with appropriate finite element equations in the subdomains. These facts on HDDM cause that conventional preconditioners such as the incomplete Cholesky decompostion cannot be applied into the iterative procedure. BDD originally proposed by Mandel in [3] is the appropriate preconditioner for HDDM, and succeeds in applying into HDDM for structural problems; see, for example, Ogino, et al. [4].

The coarse grid problem to balance indeterminate DOF on subdomain problems plays a key role in BDD. In case of 3-D structural problems, the number of DOF of the coarse grid problem is equal to $6 \times K$, where K denotes the number of subdomains. However, in case of 3-D magnetostatic problems, its number of DOF is equal to $N_1 + N_2 + \cdots + N_K$, where N_i $(i = 1, 2, \ldots, K)$ denote the numbers of DOF of the conventional piecewise linear tetrahedral finite ement space corresponding to the ith subdomain. This fact implies that it takes much more time to solve the coarse grid problem in case of the 3-D magnetostatic problem.

In this talk, to avoid raising the computational costs, VE (see, for example, Beirão da Veiga [1]) is incorporated into HDDM. In case of 3-D magnetostatic problems, indeterminated DOF consists of the conventional piecewise linear tetrahedral finite ement space among the subdomains. Now the indeterminate DOF is approximated by VE in each subdomain. Owing to the approximation of the indeterminate DOF by VE, it is expected to reduce the computational costs as well as to accept polyhedral interfaces. In the presentation, some mathematical/numerical results are shown.

Keywords: balancing decomposition method, preconditioner, virtual element, magnetostatic problem, hierarichical domain decomposition method

Mathematics Subject Classifications (2010): 65N55, 35M12, 83C50, 78M10

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