A CONTINUOUS ENERGY-BASED IMMERSED BOUNDARY METHOD FOR ELASTIC SHELLS

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ABSTRACT. The immersed boundary method is a mathematical formulation and numerical method for solving fluid-structure interaction problems. For many biological problems, such as models that include the cell membrane, the immersed structure is a two-dimensional infinitely thin elastic shell immersed in an incompressible viscous fluid. When the shell is modeled as a hyperelastic material, forces can be computed by taking the variational derivative of an energy density functional. A new method for computing a continuous force function on the entire surface of the shell is presented, where the surface is represented by spherical harmonic interpolants [1]. In this case, an analytical expression for the elastic force density can be obtained. Surface representation in the IB method was described in [3] for the case of surface tension and fiber elasticity. The method presented here considers the more general case of computing forces through variational derivatives. The new method is compared to a previous formulation where the surface and energy functional are discretized before forces are computed [2]. For the case of Stokes flow, a method for computing quadrature weights is provided to ensure the integral of the elastic spread force density remains zero throughout a dynamic simulation. Tests on the method are conducted and show that it yields more accurate force computations than previous formulations as well as more accurate geometric information such as mean curvature. The method is then applied to a model of a red blood cell in capillary flow and a 3D model of cellular blebbing.

Keywords: fluid-structure interaction, Stokes flow, hyperelasticity, red blood cells, blebbing

Mathematics Subject Classifications (2010): 74F10, 92C10, 92C37

References

[1] O. Maxian, Andrew T. Kassen, and W. Strychalski. A continuous energy-based immersed boundary method for elastic shells. *Journal of Computational Physics*, 371(15):333-362, 2018.
[2] T. G. Fai, B. E. Griffith, Y. Mori, and C. S. Peskin, Immersed boundary method for variable viscosity and variable density problems using fast constant-coefficient linear solvers I: Numerical method and results SIAM *Journal on Scientific Computing*, 35(5):B1132–B1161, 2013.
[3] V. Shankar, G. B. Wright, A. L. Fogelson, R. Kirby, A study of different modeling choices for simulating platelets within the immersed boundary method *Applied Numerical Mathematics*, 63:58-77, 2013.

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