# STABILIZED FINITE ELEMENT SCHEMES FOR PDE'S IN TIME-DEPENDENT DOMAINS

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Abstract. Problems governed by partial differential equations (PDEs) in deforming domains are of fundamental importance in science and engineering. However, developing a numerical scheme for such problems is still very challenging even when the deformation of the boundary of the domain is prescribed a priori. The possibility of excessive mesh distortion is one of the major challenges when solving such problems with numerical methods using boundary fitted meshes. The arbitrary Lagrangian-Eulerian (ALE) approach is a way to overcome this difficulty.

Standard Galerkin approach induces spurious oscillations in the numerical solution of convection dominated problems. To suppress the oscillations and to enhance the stability of the numerical scheme, stabilization methods are used for these problems. The additional stabilization in stabilized numerical schemes mainly depends on the convection term in the equation. Moreover, the numerical method that we use to incorporate the domain movement modifies the convection term in the model equation.

The stabilized finite element approximation of the transient convection dominated problem defined in time-dependent domains is considered in this work. The considered equation is designed within the framework of arbitrary Lagrangian-Eulerian (ALE) formulation. In the first part, Streamline Upwind Petrov-Galerkin (SUPG) stabilization with conservative, non-conservative ALE formulation is proposed[1, 2]. The first order backward Euler and the second order Crank-Nicolson, backward-difference methods are used for the temporal discretizations. It is shown that the stability of the semi-discrete ALE-SUPG equation is independent of the mesh velocity, whereas the stability of the fully discrete problem is unconditionally stable for conservative ALE with implicit Euler and is only conditionally stable for Crank-Nicolson, backward-difference time discretizations. Numerical computations supporting the theoretical results are included.

Further, the local projection stabilization (LPS) and the higher order discontinuous Galerkin (dG) time discretizations are studied for convection dominated problems[3]. We considered the Radau quadrature in time to implement it numerically. The stability and error estimates are shown for the mathematical basis of considered numerical scheme. The numerical analysis reveals that the proposed stabilized scheme with dG disretization in time is unconditionally stable. The theoretical estimates are illustrated with appropriate numerical examples. The dG(1) with appropriate Radau quadrature (to get exact integration in time) gives an unconditional stable solution while the Crank-Nicolson is only conditionally stable. Further the convergence order with dG(1) is shown. Next, to show the effectiveness of the stabilized scheme, a boundary layer problem with convection dominated case is considered.

**Keywords**: ALE-formulations, stabilization schemes, SUPG, LPS, discontinuous Galerkin in time, Radau quadrature.

## References

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