HYPERBOLICITY-PRESERVING STOCHASTIC GALERKIN SCHEMES FOR UNCERTAIN HYPERBOLIC SYSTEMS OF EQUATIONS

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Abstract. Intrusive Uncertainty Quantification methods such as stochastic Galerkin are gaining popularity, whereas the classical stochastic Galerkin approach is not ensured to preserve hyperbolicity of the underlying hyperbolic system. We present a modification of this method that uses a slope limiter to retain admissible solutions of the system, while providing high-order approximations in the physical and stochastic space.

The method preserves hyperbolicity and is therefore dedicated to nonlinear systems of hyperbolic equations where the classical SG is in general not applicable, in particular for low SG polynomial degrees. In contrast to other hyperbolicity-preserving UQ methods, such as the intrusive polynomial moment method, our modification does not need to know the entropy of the system and is notably simple to derive.

We apply the resulting scheme to the compressible Euler equations with uncertain initial states. The numerical results underline the strength of our method if discontinuities are present in the uncertainty of the solution.

Keywords: Uncertainty Quantification, Polynomial chaos, Stochastic Galerkin, Multi-Element, Slope Limiter, Hyperbolicity

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