CONVERGENCE ANALYSIS FOR THE DYNAMIC DIFFUSION METHOD APPLIED TO ADVECTION-DIFFUSION-REACTION PROBLEMS

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Abstract. This work presents existence, stability and a-priori error estimates to the nonlinear multi-scale Dynamic Diffusion (DD) method [3] when it is applied to the advection-diffusion-reaction equation. The DD method introduces a dynamic artificial diffusion onto both the resolved and unresolved scales. The key issue is to establish the subgrid velocity which is used to determine the smallest amount of artificial diffusion to dissipate the fine-scale kinetic energy such that the residual of the resolved scale solution vanishes. The artificial diffusion, which is introduced onto all scales, is locally and dynamically determined by imposing some restrictions on the resolved scale solution, which ultimately yields a parameter-free method and its consistency property. Preliminary results indicate that this multi-scale methodology outperforms some stabilized and capturing methods (CAU, SUPG and NSGS) for a variety of problems [3]. A similar strategy is developed for solving compressible flow problems in [1].

Considering an advection-diffusion-reaction problem with smooth solution and varying the diffusion coefficients values, the DD's method numerical performance is compared with the NSGS method [2] – a free parameter subgrid stabilizing method that introduces an extra stability only onto the subgrid scales. Both methods produce convergence rates in the same order as the classical stabilized finite element methods (in the $H^1(\Omega)$ semi-norm and in the $L^2(\Omega)$ norm). For all cases the DD method exhibits the most accurate approximate solution. In addition, our methodology also presents optimal convergence rates for dominant diffusive cases.

Keywords: Dynamic Diffusion method, stability analysis,a-priori error estimate, advection-diffusion-reaction equations

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