ADAPTIVE HDG FOR SEMILINEAR DIRICHLET BOUNDARY VALUE PROBLEMS IN CURVED DOMAINS

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ABSTRACT. The magnetic equilibrium in axisymmetric fusion reactors can be described in terms of the solution to a semilinear elliptic Dirichlet boundary value problem. The equation is posed in a domain with a piecewise smooth curved boundary. This corresponds to a region of the reactor where the plasma remains confined.

Previously, the authors had proposed a solution method involving a high order Hybridizable Discontinuous Galerkin (HDG) solver on an unfitted polygonal mesh embedded within the confinement region. The curvature of the domain is handled by a high order transfer scheme that sidesteps the use of curved triangulations or isoparametric mappings. As long as the gap between the computational domain and the true boundary remains of the order of the mesh parameter, this method preserves the order of convergence of the solver.

The transferring algorithm however may fail to resolve sharp gradients close to the boundaries and local refinement becomes necessary. We propose an h-adaptive method that relies on a residual type estimator on the embedded computational mesh. The refinement is driven by a combination of Dörfler marking and the constraint that the gap between the mesh and the curved boundary must remain of the order of the local mesh diameter. This results on a nested sequence of unfitted grids that "grow" towards the physical boundary as refinement progresses. This is an ongoing collaboration with Antoine Cerfon (NYU) and Manuel Solano (University of Concepción).

Keywords: Adaptive Hybridizable Discontinuous Galerkin, Curved boundary, Semilinear Elliptic Equations, Plasma Equilibrium, Grad–Shafranov.

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