SYMPLECTIC HYBRIDIZABLE DISCONTINUOUS GALERKIN METHODS FOR ELASTODYNAMICS

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Abstract. We introduce the first symplectic hybridizable discontinuous Galerkin (HDG) method for linear and nonlinear elastodynamics. We derive the fully discrete method with a space discretization based on Hamiltonian HDG schemes, the recently introduced by the authors. These HDG methods are devised by preserving, at a discrete level, the Hamiltonian structure of the linear and nonlinear elastic wave equations. We then use symplectic integrators to discretize in time, specifically, we use explicit symplectic partitioned Runge–Kutta methods. The fundamental feature of the resulting scheme is that the conservation of a discrete energy, which is nothing but a discrete version of the original Hamiltonian, is guaranteed. We present numerical experiments which indicate that the methods achieve optimal approximations in the sense that we observe the same accuracy than the one obtained in the steady-state cases (linear and nonlinear elasticity, respectively). We also present numerical examples supporting the energy-conserving properties of the method and showing their accurate behavior for long-time simulations.

Keywords: finite element methods, discontinuous Galerkin methods, hybrid/mixed methods, symplectic methods, elastodynamics, energy conservation, Hamiltonian systems

Mathematics Subject Classifications (2010): 65M60, 65M15, 65M20

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