UNCONDITIONAL STABILITY FOR MULTISTEP IMEX SCHEMES

RODOLFO RUBEN ROSALES, BENJAMIN SEIBOLD, DAVID SHIROKOFF, AND DONG ZHOU

Abstract. We devise a new class of linear IMEX schemes, with large regions of unconditionally stability. These new schemes are motivated by the need to stabilize high order ($r > 2$) semi-implicit IMEX splittings for the incompressible Navier-Stokes equations. However, they also provide a great amount of flexibility in stabilizing matrix splittings that may arise from alternating direction implicit (ADI) and adaptive mesh refinement (AMR) methods, or spatial discretizations for other partial differential equations. Here unconditionally stable implies the absence of a time step restriction, such as a CFL number or the type of restrictions diffusion or other higher order effects can cause. Hence one may use arbitrarily large time steps in the numerical time stepping of a differential equation. To characterize the new IMEX schemes we introduce an unconditional stability region $\mathcal{D}$, which plays a role analogous to that of the stability region in conventional multistep methods. We characterize the unconditional stability region for the new schemes, and provide computable quantities (such as a numerical range) that allow a user to guarantee an unconditionally stable scheme for a proposed implicit-explicit matrix splitting. Finally, we demonstrate the usefulness of the new approach with several examples including variable diffusion, advection diffusion, and the Navier-Stokes equations. The new schemes are designed for time stepping orders 1 through 5.

Keywords: Linear multistep ImEx, Unconditional stability, ImEx stability, High order time stepping, Semi-implicit backward differentiation.

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Massachusetts Institute of Technology
E-mail address: rrr@mit.edu

Temple University
E-mail address: seibold@temple.edu

New Jersey Institute of Technology
E-mail address: david.g.shirokoff@njit.edu

California State University Los Angeles
E-mail address: dzhou11@calstatela.edu