AUTOMATICAL VARIATIONALLY STABLE FINITE ELEMENT METHOD: STRONG FORMULATIONS

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Abstract. The Discontinuous Petrov-Galerkin (DPG) methods, first introduced by Leszek Demkowikz and Jay Gopalakrishnan in [1], are a family of Petrov-Galerkin methods where the test space is taken discontinuous. The main advantages of such methods are that, by construction, they enjoy stability at the discrete level, while also a reliable representation for the error is obtained on the fly, making it a suitable method for mesh-adaptivity based numerical implementations. The Automatical Variationally Stable Finite Element Method (AVS-FEM) is a subclass of DPG methods where the trial space is continuous. Differing from the *DPG Ultra weak formulations*, the introduction of additional trace variables are no longer required when enough regularity for the solutions is assumed. This implies, in particular, a considerable reduction in terms of degrees of freedom (cf. [2] and [3]). In this talk, we will briefly introduce the treatment of AVS-FEM formulations in strong form. We will show that rather than considering an integration by parts rule, regularity constraints and boundary conditions can be weakly imposed with the help of an adequate duality identity. This results in a natural and straightforwardly implementable FEM that keeps the spatial representation of the differential operators invariant. We will also show through numerical examples the efficiency of the method in extreme scenarios where the method gives stable solutions that are comparable with those obtained with classical stabilized methods.

Keywords: DPG, FEM, Mesh-Adaptivity.

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