EXPONENTIAL CONVERGENCE OF AN *hp*-FEM APPROXIMATION FOR PARABOLIC FRACTIONAL DIFFUSION

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ABSTRACT. In this talk, we present an approximation scheme for the following time dependent fractional diffusion problem on a bounded domain $\Omega \subseteq \mathbb{R}^d$:

$$u_t + (-\Delta)^s u = f, \qquad u(0) = u_0, \quad \text{and} \quad u|_{\partial\Omega} = 0$$

with a parameter $s \in (0,1)$. In order to localize the non-local operator $(-\Delta)^s$, we employ the so-called Caffarelli-Silvestre extension, which yields an equivalent $d+1$-dimensional local problem.

This kind of problem exhibits two sources of singularities, which need to be taken into account for the numerical scheme: The extended problem is degenerate for $y \to 0$, where y is the extended variable, and for $t \to 0$, the parabolic problem may suffer from startup singularities due to incompatibilities of the data with the boundary condition on u.

For discretization, we consider a *hp*-type finite element discretization in space (following the ideas of [1]) and an *hp*-DG method for the time variable (as in [2]). In the one dimensional setting (i.e., $d = 1$), we rigorously prove exponential convergence of the method without imposing a compatibility condition on the data.

Keywords: Fractional diffusion, nonlocal problems, *hp* Finite Element Method, Discontinuous Galerkin, parabolic problem

Mathematics Subject Classifications (2010): 65M60, 65M12, 65M15

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