HDG METHODS WITH INTEGRAL BOUNDARY CONDITIONS: THEORY AND APPLICATIONS

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ABSTRACT. The simulation of complex systems in nanoscale electronics, thermo-electronics, ocular biomechanics and other applications, gives rise to physical properties of some portions of the boundary Γ of the computational domain that cannot be handled with standard boundary conditions of Dirichlet, Neumann or Robin types. In order to deal with such a circumstance, we develop an integral boundary condition for a model Poisson problem written in mixed form to mathematically represent the behavior of a subset of the boundary $\Gamma_{ibc} \subset \Gamma$ where it is not possible to directly enforce the value of the scalar unknown p or of its associated normal flux $\vec{j} \cdot \vec{n}$ but: (1) the property that the restriction $p|_{\Gamma_{ibc}}$ is equal to a constant value (to be determined); and (2) the global flux of the vector field $\vec{j}$ across Γ_{ibc} is a given design target.

In view of engineering applications where gradients of the primal unknowns often represent quantity of primary interest, the mathematical characterization of properties (1) and (2) will be carried out in the context of the Hybridizable Discontinuous Galerkin (HDG) method, which retains all the advantages of DG methods and, thanks to hybridization, becomes computationally competitive with traditional continuous Galerkin approaches.

The mathematical framework will be validated by numerical experiments starting with convergence tests and then applications developed in Feel++, a high performance C++ library for solving PDEs using Galerkin methods in $\mathbb{R}^d, d = 1, 2, 3$ and on manifolds.

Keywords: Hybridizable Discontinuous Galerkin, Integral Boundary Condition, C++ Computing Platform, High Performance Computing.

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