DÖRFLER MARKING WITH MINIMAL CARDINALITY IS A LINEAR COMPLEXITY PROBLEM

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ABSTRACT. Adaptive finite element methods (AFEM) iterate the procedure *Solve-Estimate-Mark-Refine* in order to generate a sequence of locally refined meshes $(\mathcal{T}_\ell)_{\ell\in\mathbb{N}_0}$, where the degrees of freedom are chosen more carefully than for uniform mesh refinement: First, the discrete solution is computed on the given mesh $\mathcal{T}_\ell$. Then, local refinement indicators $(\eta_\ell(T))_{T\in\mathcal{T}_\ell}$ are computed. Based on these error estimators a subset $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ is marked for refinement. Finally, (at least) the marked elements are refined to obtain an improved mesh $\mathcal{T}_{\ell+1}$.

In his seminal work [1], Dörfler proposes a marking criterion, which allows to prove linear convergence of the energy error for each iteration of the AFEM algorithm. This marking criterion is commonly known as *Dörfler marking*: Given $\eta_\ell(T)$ for all $T \in \mathcal{T}_\ell$ and a marking parameter $0 < \theta \le 1$, construct a set $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ such that

$$\theta \sum_{T\in\mathcal{T}_\ell} \eta_\ell(T)^2 \le \sum_{T\in\mathcal{M}_\ell} \eta_\ell(T)^2 .$$

Later it was shown in [2] that the Dörfler marking criterion is not only sufficient to prove linear convergence, but even in some sense necessary.

Clearly, one aims for a subset $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ containing as few elements as possible, which satisfies the Dörfler marking criterion. In the best case, the set $\mathcal{M}_\ell \subseteq \mathcal{T}_\ell$ has minimal cardinality, i.e.,

$$\#\mathcal{M}_\ell = \min\{\#\mathcal{N} \colon \mathcal{N} \subseteq \mathcal{T}_\ell \text{ satisfies the Dörfler marking criterion}\} .$$

Dörfler [1] notes that sorting the refinement indicators would be sufficient to find such a set of minimal cardinality. Since sorting an array of length N requires $\mathcal{O}(N \log N)$ operations, while *Solve*, *Estimate* and *Refine* are (in principle) of linear cost, he, however, notes that sorting should be avoided. Stevenson [2] proposes a linear complexity algorithm to find a set of minimal cardinality up to some absolute factor 2, which satisfies the marking criterion.

In our talk, we propose a new algorithm for finding a set with minimal cardinality satisfying the Dörfler marking criterion. We show that this new algorithm terminates after at most $\mathcal{O}(N)$ operations. In particular, Dörfler marking with minimal cardinality is a linear complexity problem.

Keywords: Dörfler marking criterion, adaptive finite element method, optimal complexity.

Mathematics Subject Classifications (2010): 65N50, 65N30, 68Q25.

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