# UNCERTAINTY QUANTIFICATION METHODOLOGY FOR HYPERBOLIC SYSTEMS WITH APPLICATION TO BLOOD FLOW IN ARTERIES.

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ABSTRACT. We present a Stochastic Finite Volume [1] - ADER [2] (SFV-ADER) methodology for Uncertainty Quantification (UQ) in the general framework of systems of hyperbolic balance laws with uncertainty in parameters. The resulting method is weakly intrusive and has no theoretical accuracy barrier [3]. An illustration of the second-order version for the viscous Burgers equation with uncertain viscosity coefficient is first given in detail, under different deterministic initial conditions; the attainment of the theoretically expected convergence rate is demonstrated empirically; results and features of our scheme are discussed. We then extend the SFV-ADER method to a non-linear hyperbolic system with source terms that models one-dimensional (1D) blood flow in arteries [4], assuming uncertainties in a set of parameters of the problem. Results are then compared with measurements in a well-defined 1:1 experimental replica of the network of largest arteries in the human systemic circulation. For a 99%-confidence level, severe influence of parameter fluctuations is seen in pressure profiles, while secondary effects on flow rate profiles become visible, particularly in terminal branches. Results suggest that the proposed methodology could succesfully be applied to other physical problems involving hyperbolic balance laws.

**Keywords**: Uncertainty Quantification, Hyperbolic Equations, Stochastic Finite Volume Method, ADER Schemes, Blood Flow.

## References

[1] S. Tokareva, *Stochastic finite volume methods for computational uncertainty quantification in hyperbolic conservation laws*, Diss., ETH Zürich, Nr. 21498, 2013.

[2] E. F. Toro, R. C. Millington, and L. A. M. Nejad, *Towards Very HighOrder Godunov Schemes*. In Godunov Methods: Theory and Applications. Edited Review, E. F. Toro (Editor), pages 905-937. Kluwer Academic/Plenum Publishers, 2001.

[3] M. Petrella, S. Tokareva, E. F. Toro, *Uncertainty Quantification methodology for blood flow in elastic vessels.* (Under Review).

[4] K. S. Matthys , J. Alastruey, J. Peir, A. W. Khir, P. Sergers, P. R. Verdonck, K. H. Parker, S. J. Sherwin, *Pulse wave propagation in a model human arterial network: Assessment of 1-D numerical simulation against in vitro measurements*. Journal of Biomechanics. Vol. **40**, pp. 3476-3486, 2007.

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