NONCONFORMING TREFFTZ VIRTUAL ELEMENT METHOD FOR THE HELMHOLTZ PROBLEM

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Abstract. The numerical approximation of time-harmonic wave propagation problems presents intrinsic difficulties. In fact, due to the oscillatory nature of analytical solutions, standard (polynomial-based) finite element methods deliver accurate approximation only at very high computational cost. Therefore, for these problems, finite element methods based on approximation spaces made of oscillatory functions with the same frequency as the original problem have become increasingly popular, as they allow to reach the same accuracy with less degrees of freedom, as compared to standard polynomial methods. The so-called Trefftz finite element methods, which use test and trial functions that are, element by element, in the kernel of the considered differential operator, belong to this class.

For time-harmonic wave problems, Trefftz approximation spaces have been used in combination with several variational frameworks (e.g. least-squares, ultra weak variational formulation/discontinuous Galerkin formulations, Lagrange multipliers, variational theory of complex rays, wave-based methods), giving rise to different Trefftz finite element methods.

In this talk, we present a method for the the approximation of solutions to the 2D Helmholtz problem on general polygonal meshes, which combines the use of Trefftz basis functions with a nonconforming virtual element method (VEM) framework. The basis functions are defined in order to satisfy the Trefftz property, but are not not know in closed form, as typical in VEM. The degrees of freedom are associated with the mesh edges, and the interelement continuity constraints are imposed in a nonconforming sense á la Crouzeix-Raviart. Although the number of degrees of freedom in the definition of this new method is larger than that of other Trefftz methods, an edgewise orthogonalization-and-filtering procedure actually allows for the elimination of "almost" redundant basis functions, leading to a significant reduction of the number of degrees of freedom with no loss of accuracy. At the same time, this procedure has the beneficial effect of improving significantly the conditioning. Theoretical and numerical aspects of this Trefftz VEM will be discussed.

Keywords: virtual element method, Trefftz methods, nonconforming methods, Helmholtz problem, plane waves, polygonal meshes

Mathematics Subject Classifications (2010): 35J05, 65N12, 65N15, 65N30

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