A PRIORI AND A POSTERIORI ERROR ANALYSES OF A HIGH ORDER MIXED-FEM FOR STOKES FLOWS ON CURVED DOMAINS

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Abstract. In this talk we propose and analyze a high order mixed finite element method for the pseudostress-velocity formulation of the Stokes problem with Dirichlet boundary condition on a fluid domain Ω with curved boundary Γ. The method consists in approximating Ω by a polygonal subdomain D_h, with boundary Γ_h, where a high order Galerkin method is applied to approximate the solution, and on a transferring technique, based on integrating the extrapolated discrete gradient of the velocity, to approximate the Dirichlet boundary data on the computational boundary Γ_h. The associated Galerkin scheme is defined by Raviart–Thomas elements of order $k \geq 0$ for the pseudostress and discontinuous polynomials of degree k for the velocity, and its well-posedness is established by applying the classical Babuška–Brezzi theory, provided suitable hypotheses on the aforementioned integration segments. With this choice, the optimal order convergence is retained if the distance between Γ and Γ_h is at most of order of the meshsize h. Moreover, the pressure can be approximated optimally through a simple postprocessing of the discrete pseudostress. We also approximate the solution in $\mathrm{D}_h^c := \Omega \backslash \overline{\mathrm{D}}_h$ and derive the corresponding error estimates. In particular, when Γ_h is defined through a piecewise linear interpolation of Γ, we develop a residual-based a posteriori error estimator, and propose the associated adaptive algorithm for our Galerkin scheme. We prove reliability and efficiency of our estimator for the L^2-norm of the velocity, the $H(div)$-norm of the pseudostress and further computable terms involving curved segments and an element-by-element postprocessing of the velocity with enhanced accuracy. Numerical experiments illustrate the performance of the scheme, show the behaviour of adaptive algorithm and validate the theory.

Keywords: curved domain, high order, Stokes flow, pseudostress-velocity formulation, mixed finite element method, a posteriori error analysis, reliability, efficiency, adaptive algorithm

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