A MULTILAYER SHALLOW WATER MODEL FOR POLYDISPERSE SEDIMENTATION WITH SEDIMENT COMPRESSIBILITY MIXTURE VISCOSITY

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Abstract. A 3D multilayer shallow water approach to study polydisperse sedimentation in a viscous fluid is presented. The fluid is assumed to carry finely dispersed solid particles that belong to a finite number of species that differ in density and size. The definition of the density and average velocity of the mixture takes into account the densities of the solid particles and the fluid, and gives rise to a particular formulation that allows us to recover the global mass conservation and linear momentum balance laws of the mixture. In addition, the model includes the compressibility of the sediment the viscosity of the mixture. A dimensional analysis applied to the global mass conservation and linear momentum balance equations of the mixture allows us to discard the horizontal components of the compression term and the horizontal terms of the viscous stress tensor. This means that the final model is vertically consistent with the classical 1D vertical model. Numerical simulations illustrate the coupled polydisperse sedimentation and flow fields in various scenarios, including sedimentation with compression of the sediment and viscosity terms. In particular, various bottom topographies give rise to recirculation of the fluid and high solids concentrations.

Keywords: multilayer shallow water model, polydisperse sedimentation, nonconservative products, path-conservative method, viscous flow, compression of the sediment, recirculation.

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