HIGH ORDER H(curl)-CONFORMING APPROXIMATION SPACES IN CURVED MESHES FOR PHOTONIC WAVEGUIDE ANALYSIS

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ABSTRACT. A hierarchy of $\mathbf{H}$(curl)-conforming approximation spaces is applied in a high-precision scheme for the modal analysis of photonic waveguides. For the design of such devices, the propagation constant is often required to be approximated with relative error as small as 10^{-12}. Inhomogeneous media, reduced dimensions, and curved geometries are aspects that need to be taken into account in order to ensure the aforementioned requirements on the precision of the numerical method. In the chosen formulation [1], the calculations are performed over a cross section of the waveguide, with $\mathbf{H}$(curl)-conforming elements being used for the transverse components of the electric field, and H^1-conforming elements being adopted for the longitudinal component. Interface and boundary conditions of inhomogeneous waveguides are naturally enforced by imposing the continuity of tangential components of the electric field. An efficient use of sparse matrices, combined with high order convergence rates of the approximation spaces, make the scheme an appropriate choice for this problem. Applying the covariant Piola mapping, all calculations are performed in the reference element, simplifying the computations when dealing with complex geometries. The first family of Nédélec elements [2] are chosen, which have been implemented hierarchically in the `NeoPZ` framework (`https://github.com/labmec/neopz`). This aspect is particularly helpful in their integration with the *hp*-adaptive capabilities of `NeoPZ` [3]. The analysis of waveguides with increasing complexities is used to illustrate the scheme's efficiency and the obtained convergence rates. The importance of a careful geometrical representation of curved interfaces in the discretized domain is discussed and non-linear mappings are explored to recover the expected convergence rates. Finally, *hp*-adaptivity capabilities are illustrated for a given reference solution.

Keywords: finite elements, edge elements, high order elements, photonic waveguides

Mathematics Subject Classifications (2010): 78M10, 65N30, 78A50.

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