IMPLICIT HYBRIDIZED DISCONTINUOUS GALERKIN METHODS FOR COMPRESSIBLE MAGNETOHYDRODYNAMICS

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Abstract. We present implicit hybridized discontinuous Galerkin (HDG, EDG and IEDG) methods for ideal and resistive compressible magnetohydrodynamics (MHD) coupled with a generalized Lagrange multiplier (GLM) for hyperbolic-parabolic divergence cleaning. We show the optimal convergence of the schemes for two smooth, ideal MHD problems using various polynomial orders and validate our resistive implementation for a magnetic reconnection problem. For smooth problems, we show that using the Lagrange multiplier can reduce the errors in the divergence of the magnetic field by two orders of magnitude. For nonsmooth problems, we develop a physics-based shock capturing method to deal with shocks, contact discontinuities, and other unresolved features (such as strong current sheets, shear layers and thermal gradients). Our shock capturing method features the addition of physics-based artificial bulk viscosity, molecular viscosity, thermal conductivity, and electric resistivity to the MHD equations by ensuring cell Péclet number of order $\mathcal{O}(1)$. We demonstrate the robustness of our shock capturing method on a number of test cases and compare our results both qualitatively and quantitatively against other MHD solvers. For shock problems, we show that the divergence cleaning becomes an essential step in ensuring numerical stability.

Keywords: implicit hybridized discontinuous Galerkin, magnetohydrodynamics (MHD), shock-capturing methods, generalized Lagrange multiplier.

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