TIME-DISCONTINUOUS PETROV-GALERKIN METHODS FOR THE ADVECTION-DIFFUSION EQUATION

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Abstract. Variational space-time formulations have been widely employed in the last decades for the resolution of Partial Differential Equations. It is known that some low-order space-time variational formulations are equivalent to some implicit time integrators [1]. For example, the discontinuous Galerkin method using constants in time is algebraically equivalent to the Backward Euler method [2]. However, a general variational formulation of explicit methods in time is still unavailable.

In this work, we present a time-discontinuous Petrov-Galerkin formulation for the linear parabolic problems that, under some restrictions on the trial and test spaces, leads to explicit methods in time. For that, we perform a downwind (nonconforming) approximation of the solution at the time interfaces. We also build trial and test spaces for the linear advection-diffusion equation that lead to explicit Runge-Kutta methods of any stage [4]. In this construction, we define some orthogonality conditions between the trial and test basis functions to obtain an explicit method. With such restrictions, we obtain different trial and test spaces, where one of them is an incomplete space of polynomials. This approach enables us to design explicit time-domain (goal-oriented) adaptive algorithms [3].

Keywords: discontinuous-in-time Petrov-Galerkin formulations, downwind approximation, explicit Runge-Kutta methods, advection-diffusion equation

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