

# PROJECTION IN NEGATIVE NORMS AND THE REGULARIZATION OF ROUGH LINEAR FUNCTIONALS

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ABSTRACT. Rough linear functionals (such as Dirac Delta distributions) often appear on the right-hand side of variational formulations of PDEs. As they live in negative Sobolev spaces, they dramatically affect adaptive finite element procedures to approximate the solution of a given PDE.

In this talk we propose an alternative that, in a first step, computes a projection of the rough functional over piecewise polynomial spaces, up to a desired precision in a negative norm sense. The projection (being  $L^p$ -regular) is then used as the right-hand side of a regularized problem for which adaptive Galerkin methods performs better. A complete error analysis of the proposed methodology will be shown, together with numerical experiments.

**Keywords:** Rough linear functionals, best approximation, negative norms, regularization.

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