ACOUSTIC AND ELECTROMAGNETIC TRANSMISSION PROBLEMS: WAVENUMBER-EXPLICIT BOUNDS AND RESONANCE-FREE REGIONS

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Abstract. We consider the Helmholtz and the time-harmonic Maxwell transmission problems with one penetrable star-shaped Lipschitz obstacle. Under a natural assumption about the ratio of the wavenumbers, we prove bounds on the solution in terms of the data, with these bounds explicit in all parameters. For example, in the case of the Helmholtz equation, the (weighted) H^1 norm of the solution is bounded by the L^2 norm of the source term, independently of the wavenumber.

Such explicit bounds are key to developing frequency-explicit error analysis for numerical methods such as FEM and BEM. The "shape-robustness" allows to quantify how variations in the shape of the obstacle affect the solution and makes the bounds particularly suitable for uncertainty quantification (UQ) applications. These bounds then imply the existence of a resonance-free strip beneath the real axis.

The main novelty of these results is that the only comparable results currently in the literature are for smooth, convex obstacles with strictly positive curvature, while here we assume only Lipschitz regularity and star-shapedness with respect to a point. Furthermore, our bounds are obtained using identities first introduced by Morawetz (essentially integration by parts), whereas the existing bounds use the much-more sophisticated technology of microlocal analysis and propagation of singularities. We also adapt existing results to show that if the assumption on the wavenumbers is lifted, then no bound with polynomial dependence on the wavenumber is possible.

Keywords: transmission problem, resonance, Helmholtz equation, acoustics, frequency explicit, wavenumber explicit, Lipschitz domain, Morawetz identity, semiclassical analysis, Maxwell equations, electromagnetic wave

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