SCATTERING BY FRACTAL SCREENS: FUNCTIONAL ANALYSIS AND COMPUTATION

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ABSTRACT. The mathematical analysis and numerical simulation of acoustic and electromagnetic wave scattering by planar screens is a classical topic. The standard technique involves reformulating the problem as a boundary integral equation (BIE) on the screen, which can be solved numerically using a boundary element method (BEM). Theory and computation are both well-developed for the case where the screen is an open subset of the plane with smooth (e.g. Lipschitz or smoother) boundary. In this talk I will explore the case where the screen is an arbitrary subset of the plane; in particular, the screen could have fractal boundary, or itself be a fractal. Such problems are of interest in the study of fractal antennas in electrical engineering, light scattering by snowflakes/ice crystals in atmospheric physics, and in certain diffraction problems in laser optics. The roughness of the screen presents challenging questions concerning how boundary conditions should be enforced, and the appropriate function space setting. But progress is possible and there is interesting behaviour to be discovered: for example, a sound-soft screen with zero area (planar measure zero) can scatter waves provided the fractal dimension of the set is large enough. This research has also motivated investigations into the properties of fractional Sobolev spaces (the classical Bessel potential spaces) on non-Lipschitz domains. Accurate computations are also challenging because of the need to adapt the basis functions to the fine structure of the fractal. I will present a BEM convergence theory together with numerical results and outline some outstanding open questions.

Keywords: scattering, acoustic waves, screens, Helmholtz equation, boundary integral equations, non-Lipschitz sets, fractals, boundary element method, Sobolev spaces

Mathematics Subject Classifications (2010): 35Q60, 45B05, 65N38, 65R20, 78A45, 78M15

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