# A POSTERIORI ERROR ANALYSIS FOR RANDOM CONSERVATION LAWS

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Abstract. To account for uncertainties in numerical simulations and keeping track of the resulting outcomes has become an important issue in recent years. A significant aspect when considering uncertainty as an additional parameter is to quantify the errors that arise from numerical discretization. Additionally, it is desirable to distinguish between errors from discretizing the random space and deterministic errors that arise from spatio-temporal discretization.

To address this issue we present an a posteriori error analysis for random hyperbolic conservation laws with uncertain initial condition and random flux function, where the uncertainty is modeled as random variable. In this talk we focus on the Stochastic Collocation method to discretize the random space. The physical space is then discretized by a Discontinuous Galerkin scheme. The relative entropy stability framework of Dafermos [1] combined with the reconstruction of the numerical solution [2] allows us to derive a computable a posteriori error estimator for the random conservation law. It turns out that the corresponding residual admits a decomposition into a spatial and a stochastic residual, which enables us to control the errors arising from spatial and stochastic discretization. Moreover, this splitting gives rise to new adaptive numerical schemes where the spatial and stochastic residual are used as indicators for the local refinement. Besides the theoretical findings we present numerical computations for the Euler equations.

**Keywords**: hyperbolic conservation laws, random pdes, a posteriori error estimates, stochastic collocation method, discontinuous Galerkin method, adaptive mesh refinment

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## References

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