NUMERICAL SOLUTION OF TIME FRACTIONAL ADVECTION-DISPERSION EQUATIONS

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Abstract. We consider time fractional advection-dispersion equations with variable coefficients and fractional derivatives in the sense of Caputo. These equations are useful in a variety of fields, including fluid flow in porous media and population dynamics ([4, 5]). The first part of the talk is devoted to the stable solution of a linear two dimensional problem by an implicit finite difference scheme ([3]). The second part deals with a one dimensional problem with a nonlinear forcing term. The method of solution is inspired by the two dimensional scheme mentioned above but the approximation of the nonlinear term requires additional hypotheses of the type Lipschitz condition. In both cases we prove stability and convergence of the numerical scheme. Illustrative numerical experiments are included and there are some final remarks related to applications of the methods in a variety of situations, for instance, in the numerical solution of inverse problems ([2]) and time fractional Fisher's equation of population dynamics ([1]).

Keywords: Caputo fractional derivative, advection-dispersion equation, variable coefficients, nonlinear source term.

Mathematics Subject Classifications (2010): 35R11, 65M06, 65M32.

References

[1] M. Alquran, K. Al-Khaled, T. Sardar and J. Chattopadhyay. Revisited Fisher's equation in a new outlook: A fractional derivative approach. Physica A 438:81–93, 2015.

[2] M.D. Echeverry and C.E. Mejía. A two dimensional discrete mollification operator and the numerical solution of an inverse source problem. Submitted, 2018.

[3] C.E. Mejía and A. Piedrahita. A finite difference approximation of a two dimensional time fractional advection-dispersion problem. ArXiv:1807.07393v1.

[4] S. Momani. An algorithm for solving the fractional convection-diffusion equation with nonlinear source term. Communications in Nonlinear Science and Numerical Simulation 12:1283–1290, 2007.

[5] P. Zhuang, F. Liu, V. Anh, and I. Turner. Numerical methods for the variable-order fractional advection-diffusion equation with a nonlinear source term. SIAM *Journal on Numerical Analysis*, 47(3):1760–1781, 2009.

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