A FILTERED STOCHASTIC GALERKIN METHOD FOR HYPERBOLIC EQUATIONS

MARTIN FRANK, JONAS KUSCH, AND RYAN G. MCCLARREN

ABSTRACT. Uncertainty Quantification for nonlinear hyperbolic problems becomes a challenging task in the vicinity of shocks. Standard intrusive methods such as Stochastic Galerkin suffer from Gibb's phenomenon, i.e. the solution exhibits oscillations in the random space. These oscillations yield non-physical, discontinuous moment approximations and can lead to non-hyperbolic moment systems [1].

In order to mitigate oscillations, we filter the expansion coefficients of the Stochastic Galerkin solution. Filters are a common tool to dampen oscillations in various research areas, see e.g. [2]. Their main challenge lies in the choice of an adequate filter strength, which mitigates oscillations while preserving a certain solution accuracy. The talk focuses on a Lasso regression filter introduced in [3], which yields a sparse, non-oscillatory solution representation. We use this sparsity to adaptively pick the filter strength such that the moment with highest order is set to zero. This automated choice of the filter strength avoids the tedious task of picking a suitable filter parameter and at the same time minimizes the filtered distance to the exact solution. Furthermore, the simplicity of the filter allows an easy integration into existing Stochastic Galerkin code frameworks.

We demonstrate the effectiveness of our method by investigating Burgers' and the Euler equations. Results show a reduction of oscillations at shocks, which leads to an improved approximation of expectation values and the variance compared to Stochastic Galerkin.

Keywords: uncertainty quantification, conservation laws, hyperbolic, intrusive, oscillations, filter, Lasso regression

Mathematics Subject Classifications (2010): 35L65, 35R60, 60G35

REFERENCES

[1] Poëtte, Gaël and Després, Bruno and Lucor, Didier. Uncertainty quantification for systems of conservation laws. Elsevier *Journal of Computational Physics*, 228(7):2443–2467, 2009.

[2] McClarren, Ryan G and Hauck, Cory D Robust and accurate filtered spherical harmonics expansions for radiative transfer. Elsevier *Journal of Computational Physics*, 229(16):5597–5614, 2010.

[3] Kusch, Jonas and McClarren, Ryan G and Frank, Martin. Filtered Stochastic Galerkin Methods For Hyperbolic Equations. arXiv preprint arXiv:1808.00819, 2018.

KARLSRUHE INSTITUTE OF TECHNOLOGY
E-mail address: martin.frank@kit.edu

KARLSRUHE INSTITUTE OF TECHNOLOGY
E-mail address: jonas.kusch@kit.edu

UNIVERSITY OF NOTRE DAME
E-mail address: rmcclarr@nd.edu