CONSERVATIVE DISCONTINUOUS FINITE VOLUME AND MIXED SCHEMES FOR A NEW FOUR-FIELD FORMULATION IN POROELASTICITY

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Abstract. We introduce a hybrid numerical method for the approximation of linear poroelasticity equations, representing the interaction between the non-viscous filtration flow of a fluid and the linear mechanical response of a porous medium. In the proposed formulation, the primary variables in the system are the solid displacement, the fluid pressure, the fluid flux, and the total pressure. A discontinuous finite volume method is designed for the approximation of solid displacement using a dual mesh, whereas a mixed approach is employed to approximate fluid flux and the two pressures. We focus on the stationary case and the resulting discrete problem exhibits a double saddle-point structure. Its solvability and stability are established in terms of bounds that do not depend on the modulus of dilation of the solid. We derive optimal error estimates in suitable norms, for all field variables; and we exemplify the convergence and locking-free properties of this scheme through a series of numerical tests.

Keywords: Biot consolidation problem; Discontinuous finite volume methods; Mixed finite elements; Locking-free approximations; Conservative schemes; Error estimates.

Mathematics Subject Classifications (2010): 65N30; 76S05; 74F10; 65N15.

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