UNCERTAINTY QUANTIFICATION FOR STRONGLY DEGENERATE PARABOLIC EQUATIONS MODELLING SEDIMENTATION

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ABSTRACT. In this presentation we consider numerical methods for the quantification of the stochastic variability of solutions $u = u(x, t)$ of the strongly degenerate parabolic equation

$$\partial_t u + \partial_x f(u) = \partial_x^2 A(u), \quad (x, t) \in I \times (0, T), \quad T > 0,$$

defined on the interval $I$ with suitable initial and boundary conditions. The uncertainty arises from uncertainty in the parameters that define the function $a = a(u)$, where

$$A(u) = \int_0^u a(s) \, ds, \quad a \in L^1[0, u_{\text{max}}], \quad a(u) \geq 0 \quad \text{for} \ 0 \leq u \leq u_{\text{max}}.$$

Under the assumption of strong degeneracy, the equation (1) arises in a number of applications, including a model of sedimentation of flocculated suspensions [3]. It is frequently assumed that

$$a(u) \begin{cases} = 0 & \text{for} \ u \leq u_c \ \text{and} \ u > u_{\text{max}}, \\ > 0 & \text{for} \ u_c < u < u_{\text{max}}, \\ \geq 0 & \text{for} \ u = u_{\text{max}}, \end{cases}$$

where $u_c \geq 0$ is a given critical value, so that (1) degenerates wherever $u \leq u_c$.

The hybrid stochastic Galerkin (HSG) method is an intrusive stochastic Galerkin (SG) discretization method that was successfully applied to several non-linear PDEs [1, 4]. The idea of intrusive SG discretizations is to transform the underlying PDE, which is assumed to depend on random parameters, into a deterministic system by means of a Galerkin projection onto the stochastic space. We present an appropriate numerical scheme, which is based on central upwind method [6], and apply it to several examples motivated by real-world applications.

Keywords: Clarifier-thickener model, polynomial chaos, uncertainty quantification, hybrid stochastic Galerkin, finite volume method

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REFERENCES


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