UNCERTAINTY QUANTIFICATION FOR STRONGLY DEGENERATE PARABOLIC EQUATIONS MODELLING SEDIMENTATION

RAIMUND BÜRGER AND ILJA KRÖKER

ABSTRACT. In this presentation we consider numerical methods for the quantification of the stochastic variability of solutions $u = u(x,t)$ of the strongly degenerate parabolic equation

$$\partial_t u + \partial_x f(u) = \partial_x^2 A(u), \quad (x,t) \in I \times (0,T), \quad T > 0, \tag{1}$$

defined on the interval I with suitable initial and boundary conditions. The uncertainty arises from uncertainty in the parameters that define the function $a = a(u)$, where

$$A(u) = \int_0^u a(s)\,\mathrm{d}s, \quad a \in L^1[0, u_{\max}], \quad a(u) \geq 0 \quad \text{for } 0 \leq u \leq u_{\max}.$$

Under the assumption of strong degeneracy, the equation (1) arises in a number of applications, including a model of sedimentation of flocculated suspensions [3]. It is frequently assumed that

$$a(u) \begin{cases} = 0 & \text{for } u \leq u_{\mathrm{c}} \text{ and } u > u_{\max}, \\ > 0 & \text{for } u_{\mathrm{c}} < u < u_{\max}, \\ \geq 0 & \text{for } u = u_{\max}, \end{cases}$$

where $u_{\mathrm{c}} \geq 0$ is a given critical value, so that (1) degenerates wherever $u \leq u_{\mathrm{c}}$.

The hybrid stochastic Galerkin (HSG) method is an intrusive stochastic Galerkin (SG) discretization method that was successfully applied to several non-linear PDEs [1, 4]. The idea of intrusive SG discretizations is to transform the underlying PDE, which is assumed to depend on random parameters, into a deterministic system by means of a Galerkin projection onto the stochastic space. We present an appropriate numerical scheme, which is based on central upwind method [6], and apply it to several examples motivated by real-world applications.

Keywords: Clarifier-thickener model, polynomial chaos, uncertainty quantification, hybrid stochastic Galerkin, finite volume method

Mathematics Subject Classifications (2010): 35R60, 65M08, 68U20

REFERENCES

[1] A. Barth, R. Bürger, I. Kröker and C. Rohde. Computational uncertainty quantification for a clarifier-thickener model with several random perturbations: A hybrid stochastic Galerkin approach. *Computers & Chemical Engineering*, 89: 11-26, 2016.

[2] R. Bürger and S. Diehl. Convexity-preserving flux identification for scalar conservation laws modelling sedimentation. *Inverse Problems*, 29(4):045008, 30, 2013.

[3] R. Bürger, K. H. Karlsen, and J. D. Towers. A model of continuous sedimentation of flocculated suspensions in clarifier-thickener units. *SIAM J. Appl. Math.*, 65(3):882–940, 2005.

[4] R. Bürger and I. Kröker. Hybrid Stochastic Galerkin Finite Volumes for the Diffusively Corrected Lighthill-Whitham-Richards Traffic Model. *Springer Proceedings in Mathematics and Statistics*, 200:189 - 197, 2017.

[5] R. Bürger and I. Kröker. Computational uncertainty quantification for some strongly degenerate parabolic convection-diffusion equation. *Journal of Computational and Applied Mathematics*, 348:490 - 508, 2019.

[6] A. Kurganov and G. Petrova. Central-upwind schemes on triangular grids for hyperbolic systems of conservation laws. *Numer. Methods Partial Differential Equations*, 21(3):536 - 552, 2005.

CI²MA AND DEPARTAMENTO DE INGENIERÍA MATEMÁTICA, FACULTAD DE CIENCIAS FÍSICAS Y MATEMÁTICAS, UNIVERSIDAD DE CONCEPCIÓN, CASILLA 160-C, CONCEPCIÓN, CHILE
E-mail address: rburger@ing-mat.udec.cl

IANS, UNIVERSITÄT STUTTGART, ALLMANDRING 5B, D-70569 STUTTGART, GERMANY
E-mail address: ikroeker@mathematik.uni-stuttgart.de