

THE INVERSE OF A FINITE ELEMENT DISCRETIZATION OF THE FRACTIONAL LAPLACIAN CAN BE APPROXIMATED BY \mathcal{H} -MATRICES

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ABSTRACT. Since the pioneering work [2] of Caffarelli and Silvestre, it is known that the fractional Laplace operator $(-\Delta)^s$ for $s \in (0, 1)$ can be represented as the Dirichlet-to-Neumann map of a degenerate PDE on an unbounded domain. Indeed, the fractional Laplacian is a non-local operator, and it can be represented as an integral operator with a singular kernel [1]. Hence, we can expect that techniques used in boundary element methods can be applied also in this case. For example, it is known that matrices arising in discretizations of Galerkin boundary element methods can be approximated by Hierarchical matrices [4]. Even more, the same is true for their inverses [3]. In this talk, we show how these results and techniques carry over to the case of the fractional Laplace operator.

Keywords: Fractional PDE, Integral operators, Hierarchical matrices

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