WAVE PROPAGATION IN HYPERBOLIC METAMATERIALS: FREQUENCY DOMAIN

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Abstract. We consider the wave propagation in 2D hyperbolic metamaterials [1, 2], which is described by the Maxwell's equations with a diagonal anisotropic tensor of dielectric permittivity, behaving like the Drude material in one of the directions, i.e. $\varepsilon(\omega) = \text{diag}(\varepsilon_{||}, \varepsilon_{\perp})$, $\varepsilon_{||} = 1$, $\varepsilon_{\perp} = 1 - \Omega_p^2\omega^{-2}$). The corresponding time-domain problem is well-posed and stable [3]. For every $\omega \in \mathbb{R}$, either of the following regimes of the wave propagation is realized: waves propagate forward in the directions $||$, $\perp$ (when $|\omega| > \Omega_p$) or they propagate backward in the direction $||$ and forward in the direction $\perp$ (corresponding to the frequencies $|\omega| < \Omega_p$). In the frequency domain, for $|\omega| > \Omega_p$, the corresponding problem reduces to the Helmholtz equation, however, for $0 < |\omega| < \Omega_p$, this PDE becomes hyperbolic (Klein-Gordon equation).

We concentrate on this latter case. In order to show the well-posedness of this problem, we demonstrate the validity of the limiting absorption principle. An interesting outcome of this analysis is that the respective fundamental solution no longer vanishes outside of the propagation cone (as it is the case for the fundamental solution for the initial-value problem), but features a certain anisotropic exponential decay there.

Moreover, we introduce an appropriate radiation condition (resembling the angular spectrum representation radiation condition [4]) and prove the existence/uniqueness of the solution in the appropriate spaces. Compared to the elliptic case, for the same data (right-hand side) the solutions possess less regularity (for L^2_{loc} data, the solution is in H^1_{loc} rather than in H^2_{loc}), and the singularities concentrate along the characteristic curves.

We will accompany the discussion with numerical experiments, performed using the adapted PMLs of [5], that illustrate our theoretical findings.

Keywords: Maxwell equations, hyperbolic metamaterials, limiting absorption principle, limiting amplitude principle, radiation condition

Mathematics Subject Classifications (2010): 35Q61,35L55

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