TRANSPARENT BOUNDARY CONDITIONS FOR THE WAVE PROPAGATION IN FRACTAL TREES

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ABSTRACT. In this work we consider the problem of the sound propagation in a bronchial network of the human lung. Asymptotically, this phenomenon can be modelled by a weighted wave equation posed on a fractal (i.e. self-similar) 1D tree [1]. The main difficulty for the numerical resolution of the problem is the 'infiniteness' of the geometry. To deal with this issue, we present transparent boundary conditions, used to truncate the computational domain to a finite subtree. The construction of such transparent conditions relies on the approximation of the Dirichlet-to-Neumann (DtN) operator.

We consider two types of boundary conditions: Dirichlet and Neumann. These boundary conditions incorporated into the original model via an appropriate variational framework. The respective DtN operators are convolution operators, whose symbols are meromorphic functions that satisfy a certain non-linear functional equation. We present two approaches to approximate the DtN in the time domain, alternative to the low-order absorbing boundary conditions [1].

The first approach stems from the use of the convolution quadrature [2, 3], which consists in constructing an exact DtN for the problem semi-discretized in time. In this case the combination of the explicit leapfrog method for the volumic terms and the implicit trapezoid rule for the boundary terms leads to a second order scheme stable under the classical CFL condition. The second method is motivated by the Engquist-Majda ABCs [4], and consists in approximating the DtN by local operators, obtained from the truncation of the meromorphic series which represents the symbol of the DtN. We show how the respective error can be controlled and provide the complexity estimates.

We finish the discussion with numerical experiments, comparing the efficiency of the presented methods.

Keywords: wave propagation, absorbing boundary conditions, fractal trees, quantum graphs

Mathematics Subject Classifications (2010): 65M12, 65M15, 34L15

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