

LOG ORTHOGONAL FUNCTIONS: THEIR APPROXIMATION PROPERTIES AND APPLICATIONS TO FRACTIONAL DIFFERENTIAL EQUATIONS

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ABSTRACT. We present two new classes of orthogonal functions, log orthogonal functions (LOFs) and generalized log orthogonal functions (GLOFs), which are constructed by applying a log mapping to the Laguerre polynomials. We develop basic approximation theory for these new orthogonal functions and apply them to several typical fractional differential equations whose solutions exhibit weak singularities. Our error analysis and numerical results show that our methods based on the new orthogonal functions are particularly suitable for functions which have weak singularities at one endpoint, and can lead to exponential convergent rates, as opposed to low algebraic rates if usual orthogonal polynomials are used.

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