LAX–WENDROFF TYPE RECONSTRUCTION-FREE HIGH-ORDER SHOCK-CAPTURING FINITE DIFFERENCE METHODS FOR HYPERBOLIC CONSERVATION LAWS

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ABSTRACT. We present an order-adaptative finite difference numerical method for systems of conservation laws. The method uses centered $2p+1$ stencils, where p may take values in $\{1, 2, \ldots, p_{max}\}$ according to a family of smooth indicators in the stencils. For linear problems, the method writes as a combination between a robust first order numerical method and the $2p$-order generalized Lax–Wendroff methods, so that the numerical method is first order near shocks and of order $2p$ in smooth regions, where $2p+1$ is the size of the biggest stencil in which large gradients are not detected. An algorithm to compute the coefficients of the method, the choice of the smooth indicators, and the stability analysis (based on [2]) will be presented.

For nonlinear problems, the original LW procedure requires the conversion of the time derivatives to spatial derivatives through the so-called Cauchy–Kovalevskaya process, what may increase dramatically the computational cost (see [3]). To avoid this, we adapt the strategy of the Approximate Taylor methods introduced in [4]. The main difference is that, unlike [4], a WENO flux reconstruction (see [1]) is not needed to ensure the stability of the method, what reduces significantly the computational cost. The general structure of the method and some numerical tests will be shown, in which the results are compared with those provided by standard WENO and Approximate Taylor methods.

Keywords: Lax–Wendroff methods, hyperbolic conservation laws.

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