## ON THE NUMERICAL SOLUTION OF DUAL-PHASE-LAGGING EQUATION IN A MULTILAYERED NANOSCALE SOLID

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ABSTRACT. In this article we consider the dual-phase-lagging heat conduction equation in a multi-layered nano-scale solid structure with a temperature-jump boundary condition and a new thermal lagging interfacial effect condition between layers. We construct a second-order accurate finite difference scheme for solving the heat conduction problem. To be more precise our numerical approximation is such that at all inner grid points has second-order temporal and spatial truncation errors, while at the boundary points and at inter-facial points has second-order temporal truncation error and first-order spatial truncation error, respectively. Then, we prove that the difference scheme introduced is unconditionally stable, convergent, and has rate of convergence two in space and time for the  $L_{\infty}$ -norm. Moreover, we give two numerical examples to confirm our theoretical results and the application to the thermal analysis in the case of a three-layered problem.

Keywords: convergence; lagging equation; finite difference scheme; stability Mathematics Subject Classifications (2010): 65M06, 65M12, 76A20

## References

- H. Sun, Z-Z Sun, and W. Dai. A second-order finite difference scheme for solving the dual-phase-lagging equation in a double-layered nano-scale thin film. *Numer. Methods Partial Differential Equations* 33(1):142– 173, 2017.
- [2] T. Q. Qiu, and C. L. Tien. Femtosecond laser heating of multi-layer metals-I. Analysis, Int J Heat Mass Transfer 37:2789–2797, 1992.
- [3] T.W. Tsai, and Y. M. Lee. Analysis of micro-scale heat transfer and ultra fast thermoelasticity in a multilayered metal film with nonlinear thermal boundary resistance. *Int J Heat Mass Transfer* 62:87–98, 2013.
- [4] Y. Mao, and M. Xu. Lattice Boltzmann numerical analysis of heat transfer in nano-scale silicon films induced by ultra-fast laser heating, Int J Thermal Sci 89:210–221, 2015.

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