SHAPE HOLOMORPHY OF THE BOUNDARY INTEGRAL OPERATORS IN ACOUSTIC WAVE SCATTERING: APPLICATIONS TO FORWARD AND INVERSE UNCERTAINTY QUANTIFICATION

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Abstract. Partial differential equations (PDEs) have been extensively used to model complex processes and phenomena in a wide range of applications. These models are subject to sources of uncertainty, whose effects we would like to characterize and quantify. Computational Uncertainty Quantification (UQ) is the field of research aiming at understanding how this fluctuations propagate throughout a model. The numerical approximation of these phenomena becomes a challenge whenever the number of parameters describing the sources of uncertainty is large or even infinity, due to the so–called *curse of dimensionality*.

Recently in [1], a principle to construct sparse representations of solution manifolds for parametric PDEs has been established. This approach relies upon the study of the smooth or, more precisely, the *holomorphic* dependence of the solution of a PDE or other *Quantity of Interest* (QoI) on the set of parameters describing the source of uncertainty in the mathematical model.

In this talk we discuss the holomorphic dependence of the boundary integral operators (BIOs), as well as the solution of boundary integral equatios (BIEs), in the shape of the boundary on which the BIOs and the BIEs are posed, under the assumption that the boundary is of class C^2 in two dimensions. This property, also known as *shape holomorphy*, has been studied in the context of volume formulations for Helmholtz, Maxwell and Navier-Stokes equations [2, 3, 4]. However, to our knowledge, no work in this subject has been done in the context of BIOs and BIEs for acoustic wave scattering.

Furthermore, we discus consequences of this mathematical result for both forward and inverse UQ. Regarding the latter, in this talk we analyze the Bayesian approach to inverse problems in acoustic wave scattering, for the the approximation of the unknown shape of the scatterer, a technique known as *Bayesian shape inversion* [5, 6].

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References

[1] A. Chkifa, A. Cohen, and Ch. Schwab. Breaking the curse of dimensionality in sparse polynomial approximation of parametric PDEs. *Journal de Mathématiques Pures et Appliquées*, 103(2):400–428, 2015.

[2] R. Hiptmair, L. Scarabosio, C. Schillings, and Ch. Schwab. Large deformation shape uncertainty quantification in acoustic scattering. *Adv. Comput. Math.* (2018), https://doi.org/10.1007/s10444-018-9594-8.

[3] C. Jerez-Hanckes, Ch. Schwab, and J. Zech. Electromagnetic wave scattering by random surfaces: Shape holomorphy. *Mathematical Models and Methods in Applied Sciences*, 17(12):2229–2259, 2016.

[4] A. Cohen, Ch. Schwab, and J. Zech. Shape holomorphy of the stationary Navier-Stokes equations. *SIAM J. Math. Anal.* 50(2):1720–1752, 2018.

[5] C. Schillings and Ch. Schwab. Sparse, adaptive smolyak quadratures for Bayesian inverse problems. *Inverse Problems*, 29(6), 2013.

[6] R. N. Gantner and M. D. Peters. Higher–Order Quasi–Monte Carlo for Bayesian shape inversion. *SIAM/ASA J. Uncertainty Quantification.* 6(2):707–736, 2018.

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