A FAST SPARSE GRID BASED SPACE-TIME BOUNDARY ELEMENT METHOD FOR THE HEAT EQUATION

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ABSTRACT. Finite element discretizations of thermal layer potentials are usually based on two sequences nested of nested spaces

$$V_0^\Gamma \subset V_1^\Gamma \subset \cdots \subset V_{\ell_s}^\Gamma \subset \cdots \subset L^2(\Gamma), \quad V_0^I \subset V_1^I \subset \cdots \subset V_{\ell_t}^I \subset \cdots \subset L^2(I),$$

where superscript Γ indicates the boundary surface and I the time interval. If $\Phi_{\ell_s}^\Gamma$ and $\Phi_{\ell_t}^I$ denote bases of $V_{\ell_s}^\Gamma$ and $V_{\ell_t}^I$, respectively, then we obtain the full tensor product representation of the single layer potential V

$$\mathbf{V}_{\ell_s,\ell_t} := \langle V(\Phi_{\ell_s}^\Gamma \otimes \Phi_{\ell_t}^I), \Phi_{\ell_s}^\Gamma \otimes \Phi_{\ell_t}^I \rangle_{L^2(\Gamma\times I)}$$

While algorithms exist that can handle this matrix with essentially optimal complexity, the cost can be still prohibitive because of the high dimension of the full tensor product.

To overcome this difficulty, we consider sparse tensor product spaces [1]. To that end, we set

$$\begin{aligned} W_{\ell_s}^\Gamma &:= V_{\ell_s}^\Gamma \ominus V_{\ell_s-1}^\Gamma, \quad W_{\ell_s}^\Gamma = \mathrm{span}\Psi_{\ell_s}^\Gamma, \\ W_{\ell_t}^I &:= V_{\ell_t}^I \ominus V_{\ell_t-1}^I, \quad W_{\ell_t}^I = \mathrm{span}\Psi_{\ell_t}^I. \end{aligned}$$

Instead of a discretization in the full tensor product space we will consider a discretization in the sparse tensor product space

$$\widehat{U}_{\boldsymbol{L}} := \bigoplus_{2\ell_s+\ell_t\le 2L} W_{\ell_s}^\Gamma \otimes W_{\ell_t}^I.$$

The resulting matrix

$$\widehat{\mathbf{V}}_L := \Big[\langle V(\Psi_{\ell_s}^\Gamma \otimes \Psi_{\ell_t}^I), \Psi_{\ell_s'}^\Gamma \otimes \Psi_{\ell_t'}^I \rangle_{L^2(\Gamma\times I)}\Big]_{\substack{2\ell_s+\ell_t\le 2L \\ 2\ell_s'+\ell_t'\le 2L}}$$

has a much more complicated structure, because ansatz- and test functions are in general at different scales. By employing prolongation operators, the Toeplitz-structure in time and a fast multipole method in space, we arrive at an algorithm which computes the approximate solution in a complexity that essentially corresponds to that of the spatial discretization. Nevertheless, the convergence rate is nearly the same as in case of a traditional discretization in full tensor product spaces.

Keywords: Parabolic Boundary Integral Equation, Sparse Grid, Fast Algorithm.

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REFERENCES

[1] H.Harbrecht and J.Tausch, Fast Sparse Grid Based Space-Time Boundary Element Method for the Non-stationary Heat Equation *Numerische Mathematik*, 140(1), 239-264.

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