UNCERTAINTY QUANTIFICATION FOR ELLIPTIC PDES WITH RANDOM ANISOTROPIC DIFFUSION

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ABSTRACT. We will consider elliptic diffusion problems with an anisotropic random diffusion coefficient. More specifically, we will model diffusion in a medium comprised of very thin fibres, where the diffusion strength in fibre direction is notably different to the diffusion strength perpendicular to the fibres – thus we may describe the diffusion strength in fibre direction and the actual fibre direction by a vector field $\mathbf{V}$. Any uncertainty regarding the vector field $\mathbf{V}$ then propagates, yielding uncertainty in the diffusion coefficient and therefore also in the solution of the elliptic diffusion problem.

Using the vector field $\mathbf{V}$'s Karhunen-Loève expansion we can reformulate the elliptic diffusion problem into a parametric form with an infinite-dimensional random parameter. We then derive that the regularity of the solution's dependence on the random parameter is entirely determined by the decay of the vector field $\mathbf{V}$'s Karhunen-Loève expansion. Therefore, sophisticated quadrature methods, such as the quasi-Monte Carlo method or the anisotropic sparse grid quadrature, may be used to approximate quantities of interest, like the solution's mean or its variance. Moreover, given regularity of the elliptic diffusion problem, we can use multilevel adaptations of those quadrature methods to lessen the computation complexity when approximating the quantities of interest, while still yielding the expected rates of convergence. Numerical examples will be presented to supplement the theoretical results.

Keywords: uncertainty quantification, anisotropic diffusion, regularity, multilevel methods

Mathematics Subject Classifications (2010): 35R60, 65N30, 60H35

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