NASH STRATEGIES FOR CAUCHY-STOKES SHAPE IDENTIFICATION PROBLEMS

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Abstract. We address in the present work the delicate problem of detecting unknown cavities immersed in a stationary viscous fluid, using partial boundary measurements. The considered fluid obeys a Stokes regime, the cavities are inclusions and the boundary measurements are a single compatible pair of Dirichlet and Neumann data, available only on a partial accessible part of the whole boundary. This inverse inclusion Cauchy-Stokes problem is ill-posed for both the cavities and missing data reconstructions, and designing stable and efficient algorithms, which is our main goal, is not straightforward.

The ill-posedness is tackled by decentralization : we reformulate it as a three players Nash game, following the ideas introduced earlier in [1] to solve the Cauchy-Laplace (completion) problem. Thanks to a simple yet strong identifiability result for the Cauchy-Stokes system, it is enough to set up two Stokes BVP, then use them as state equations. The Nash game is then set between 3 players, the two first targeting the data completion while the third one targets the inclusion detection. The latter problem is formulated using a level-set approach, and we provided the third player with the level-set function as strategy, while its cost functional is of Kohn-Vogelius type.

The class of algorithms we propose apply to a broad range of ill-posed inverse problems, the involved computational apparatus being rather classical : use of descent algorithms for the different minimizations, use of adjoint state method to compute the sensitivities, and use of Finite Element methods to solve the state and adjoint state equations, as well as to update the level-sets. We used Freefem++ to implement these routines for our problem.

We present 2D numerical experiments for three different test-cases. For noise free, as well as for noisy -Cauchy data- Dirichlet measurements, we obtained satisfactory results, exhibiting very stable behavior with respect to the noise level (1%, 3%, 5%). The obtained results favor our 3-player Nash game approach to solve parameter or shape identification for Cauchy problems. Finally, our approach rises difficult theoretical questions, such as the existence, uniqueness and convergence issues for the level-set solution to an implicit necessary optimality condition and, related to the game-theoretic approach, the existence and convergence issues for the 3-player Nash equilibrium.

Keywords: Data completion, Cauchy-Stokes problem, shape identification, Nash games.

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References

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