NEARBY PRECONDITIONING FOR MULTIPLE REALISATIONS OF THE HELMHOLTZ EQUATION

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Abstract. Let $\mathbf{A}^{(j)}$, $j = 1, 2$, be the Galerkin matrices corresponding to the h-FEM discretisation of the exterior Dirichlet problem for the heterogeneous Helmholtz equations

$$\nabla \cdot (A^{(j)} \nabla u^{(j)}) + k^2 n^{(j)} u^{(j)} = -f.$$

In this work we answer the following question: How small must $\|A^{(1)} - A^{(2)}\|_{L^\infty}$ and $\|n^{(1)} - n^{(2)}\|_{L^\infty}$ be (in terms of k-dependence) for GMRES applied to either $(\mathbf{A}^{(1)})^{-1}\mathbf{A}^{(2)}$ or $\mathbf{A}^{(2)}(\mathbf{A}^{(1)})^{-1}$ to converge in a k-independent number of iterations for arbitrarily large k? (In other words, for $\mathbf{A}^{(1)}$ to be a good left- or right-preconditioner for $\mathbf{A}^{(2)}$.)

We prove that, if

$$k\|A^{(1)} - A^{(2)}\|_{L^\infty} \text{ and } k\|n^{(1)} - n^{(2)}\|_{L^\infty} \text{ are both sufficiently small} \tag{1}$$

then $\mathbf{A}^{(1)}$ is a good preconditioner for $\mathbf{A}^{(2)}$ when using *weighted* GMRES, and numerical experiments show the conditions (1) are sharp. Moreover, numerical experiments show that the conditions (1) are sharp for standard GMRES, but to prove $\mathbf{A}^{(1)}$ is a good preconditioner for $\mathbf{A}^{(2)}$ for standard GMRES we require a slightly stronger condition on $A^{(1)}$ and $A^{(2)}$ than that in (1).

Our motivation for tackling this question comes from calculations in uncertainty quantification (UQ) for the Helmholtz equation with *random* coefficients A and n. Such a calculation requires the solution of many deterministic Helmholtz problems, each with different A and n. The answer to the question above dictates to what extent a previously-calculated preconditioner of one of the Galerkin matrices can be used as a preconditioner for other Galerkin matrices. The extent to which one can reuse preconditioners reduces the cost of the UQ calculation.

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