HYBRID FINITE ELEMENT - SPECTRAL METHOD FOR THE FRACTIONAL LAPLACIAN: APPROXIMATION THEORY AND EFFICIENT SOLVER

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Abstract. A numerical scheme is presented for approximating fractional order Poisson problems in two and three dimensions. The scheme is based on reformulating the original problem posed over Ω on the extruded domain $\mathcal{C} = \Omega \times [0, \infty)$. The resulting degenerate elliptic *integer* order PDE is then approximated using a hybrid FEM-spectral scheme. Finite elements are used in the direction parallel to the problem domain Ω, and an appropriate spectral method is used in the extruded direction. The spectral part of the scheme requires that we approximate the true eigenvalues of the integer order Laplacian over Ω. We derive an a priori error estimate which takes account of the error arising from using an approximation in place of the true eigenvalues. We further present a strategy for choosing approximations of the eigenvalues based on Weyl's law and finite element discretizations of the eigenvalue problem. The system of linear algebraic equations arising from the hybrid FEM-spectral scheme is decomposed into blocks which can be solved effectively using standard iterative solvers such as multigrid and conjugate gradient. Numerical examples in two and three dimensions suggest that the approach is quasi-optimal in terms of complexity.

Keywords: fractional Laplacian, finite element-spectral method, numerical approximation

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