SPACE-TIME ADAPTIVE FINITE ELEMENTS FOR NONLOCAL PARABOLIC VARIATIONAL INEQUALITIES

HEIKO GIMPERLEIN AND JAKUB STOCEK

Abstract. Variational inequalities for time-dependent nonlocal differential equations have attracted recent interest in a wide variety of applications; parabolic obstacle problems arise in the pricing of American options with jump processes [5]. Mechanical problems naturally involve contact and friction at the boundary with surrounding materials. For nonlocal material laws, they are intensely studied in peridynamics [4], but even for local material laws boundary integral formulations give rise to nonlocal problems [1]. Friction also plays a role in nonlocal evolution equations in image processing [3].
This work [2] considers the error analysis of finite element discretizations and adaptive mesh refinement procedures for nonlocal dynamic contact and friction, both in the domain and on the boundary. For a large class of parabolic variational inequalities associated to the fractional Laplacian we obtain a priori and a posteriori error estimates and study the resulting space-time adaptive mesh-refinement procedures. Particular emphasis is placed on mixed formulations, which include the contact forces as a Lagrange multiplier. Corresponding results are presented for elliptic problems. Our numerical experiments for 2-dimensional model problems confirm the theoretical results: They indicate the efficiency of the a posteriori error estimates and illustrate the convergence properties of space-time adaptive, as well as uniform and graded discretizations.

Keywords: fractional Laplacian; variational inequality; space-time adaptivity; a posteriori error estimates; a priori error estimates; dynamic contact.

Mathematics Subject Classifications (2010): 65M60.

References

[1] Joachim Gwinner and Ernst P. Stephan. *Advanced Boundary Element Methods–Treatment of Boundary Value, Transmission and Contact Problems*. Springer, 2018.
[2] Heiko Gimperlein and Jakub Stocek. Space-time adaptive finite elements for nonlocal parabolic variational inequalities. *arXiv preprint arXiv:1810.06888*, 2018.
[3] Jan Lellmann, Konstantinos Papafitsoros, Carola-Bibiane Schönlieb, and Daniel Spector. Analysis and application of a nonlocal Hessian. *SIAM J. Imaging Sci.*, 8(4):2161–2202, 2015.
[4] Erdogan Madenci and Erkan Oterkus. *Peridynamic theory and its applications*. Springer, 2016.
[5] Peter Tankov. *Financial modelling with jump processes*, volume 2. CRC Press, 2003.

Maxwell Institute for Mathematical Sciences and Department of Mathematics, Heriot–Watt University, Edinburgh, EH14 4AS, United Kingdom
E-mail address: h.gimperlein@hw.ac.uk

Maxwell Institute for Mathematical Sciences and Department of Mathematics, Heriot–Watt University, Edinburgh, EH14 4AS, United Kingdom
E-mail address: js325@hw.ac.uk