SPACE-TIME ADAPTIVE FINITE ELEMENTS FOR NONLOCAL PARABOLIC VARIATIONAL INEQUALITIES

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Abstract. Variational inequalities for time-dependent nonlocal differential equations have attracted recent interest in a wide variety of applications; parabolic obstacle problems arise in the pricing of American options with jump processes [5]. Mechanical problems naturally involve contact and friction at the boundary with surrounding materials. For nonlocal material laws, they are intensely studied in peridynamics [4], but even for local material laws boundary integral formulations give rise to nonlocal problems [1]. Friction also plays a role in nonlocal evolution equations in image processing [3].

This work [2] considers the error analysis of finite element discretizations and adaptive mesh refinement procedures for nonlocal dynamic contact and friction, both in the domain and on the boundary. For a large class of parabolic variational inequalities associated to the fractional Laplacian we obtain a priori and a posteriori error estimates and study the resulting space-time adaptive mesh-refinement procedures. Particular emphasis is placed on mixed formulations, which include the contact forces as a Lagrange multiplier. Corresponding results are presented for elliptic problems. Our numerical experiments for 2-dimensional model problems confirm the theoretical results: They indicate the efficiency of the a posteriori error estimates and illustrate the convergence properties of space-time adaptive, as well as uniform and graded discretizations.

Keywords: fractional Laplacian; variational inequality; space-time adaptivity; a posteriori error estimates; a priori error estimates; dynamic contact.

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References


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