# HYBRIDIZABLE DISCONTINUOUS GALERKIN FORMULATION BASED ON VOIGT NOTATION FOR COMPUTATIONAL MECHANICS

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ABSTRACT. Discontinuous Galerkin (DG) methods have experienced an increasing success in recent years owing to their flexibility to construct high-order and non-uniform degree approximations, to develop efficient parallel implementations and to devise accurate stabilizations in convection-dominated flow problems. More recently, the hybridizable discontinuous Galerkin (HDG) framework has been proposed, starting from a mixed DG formulation and exploiting hybridization to reduce the overall number of degrees of freedom of the problem [1]. Nonetheless, classical hybridizable discontinuous Galerkin (HDG) approaches [2, 3] are known to experience suboptimal behavior when low-order approximations of symmetric second-order tensors are involved. This is especially critical in computational mechanics, where the symmetry of the Cauchy stress tensor represents the enforcement of the balance of angular momentum. In this talk, a novel HDG formulation exploiting Voigt notation [4] to enforce the symmetry of the stress tensor pointwise in each element of the mesh is discussed [5, 6]. Using equal-order polynomials of degree $k$ for all the variables, the HDG-Voigt formulation retrieves optimal convergence of order $k{+}1$ of the discretized strain rate tensor $\boldsymbol{L}^h \approx -(\boldsymbol{\nabla u} + \boldsymbol{\nabla u}^T)/2$ even for low-order approximations. Hence, a superconvergent solution $\boldsymbol{u}^\star$ can be constructed via a local postprocessing procedure and exploited to drive a degree adaptivity strategy [7]. Numerical results highlighting the optimal convergence and superconvergence properties of the high-order HDG-Voigt method will be presented, for incompressible flow and linear elasticity problems, in two and three dimensions and for different mesh elements [5, 6]. Moreover, a novel fast low-order discretization method, named face-centered finite volume (FCFV), will be introduced [8, 9]. FCFV provides locking-free solutions with no loss of accuracy for nearly incompressible materials and in the incompressible limit, features first-order convergence of fluxes/stresses without the need to perform flux reconstruction and is robust to mesh deformation and stretching.

**Keywords**: hybridizable discontinuous Galerkin, face-centered finite volume, Voigt notation, incompressible flow, linear elasticity, locking-free, superconvergence

**Mathematics Subject Classifications (2010)**: 65M60, 76D07, 76M10

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