# A CONSERVATIVE MATHEMATICAL MODEL AND NUMERICAL SOLUTION OF THE CENTRIFUGAL SETTLING WITH COALESCENCE OF LIQUID-LIQUID DISPERSIONS

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Abstract. The centrifugal sedimentation with coalescence of a dispersion of two immiscible liquids with a continuous droplet size distribution can be modeled by a population balance model with the independent variables time $t$, radius $r$ and droplet volume $v$, in the form of an integro-partial differential equation (IPDE). This IPDE is projected onto a system of partial differential equations by discretizing the droplet volume. This is accomplished by using the fixed-pivot technique of Kumar and Ramkrishna (1996a) handling any two integral properties associated with the population number density such as dispersed phase (droplets) volume fraction and droplets number concentration. Because there is loss of mass in the system when the daughter droplets are greater than the larger species considered for the numerical solution, two terms that produce the conservation of the mass or total volume of the dispersed phase are incorporated in the system of PDEs. The resulting system of partial differential equations (PDEs) is split into two systems, of homogeneous PDEs and ordinary differential equations (ODEs). The homogeneous PDEs and the ODEs are discretized using a first-order central differencing scheme and a second-order two-stage method, respectively. The model predicts the radial variation of the composition of the disperse phase layer forming at the inner wall or the outer wall of the centrifuge.

**Keywords**: Liquid-liquid separation; Mathematical modelling; Simulation; Hindered settling; Dispersion; Population balance; Coalescence.

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